- DRAFT -

Uncertainty: What Does it Mean for Ballistic Chronographs?

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ABSTRACT

Two of the most cited performance properties of ballistic armour systems that characterize resistance to penetration are the ballistic limit (V_{50}) and the proof velocity (V_{proof}). It is therefore critical that the test velocities are measured with a high degree of certainty for gaining confidence in the ballistic performance assessment for research and development, demonstration of compliance, and quality control purposes (i.e., ISO 17025). Unfortunately, common ballistic testing standards (NIJ 0101.06, ASTM E3062, etc.) do not provide sufficient clarity regarding contributors to consider, coverage factors, or interpretation of stated accuracy limits, leaving significant room for interpretation. This work applies principles of the *Guide to the expression of uncertainty in measurement* (GUM) to a COTS ballistic chronograph (SpeedTube™) to form a basis for discussions regarding interpretation of standards and the need for standards to use language consistent with the International Vocabulary of Metrology (VIM) for the benefit of researchers, manufacturers, and test facilities.

INTRODUCTON

Two of the most cited performance properties of ballistic armour systems (e.g., helmets and vests) that characterize the resistance to penetration are the ballistic limit (V_{50}) and the proof velocity (V_{proof}). It is therefore critical that the test velocities are measured with a high degree of certainty for gaining confidence in the ballistic performance assessment for research and development, demonstration of compliance, and quality control purposes. Measurement certainty is well addressed within quality management processes, such as in ISO 17025 [1], and is intended to account for variability encountered during the measurement process and includes the identification of the error sources and degree of variability of the measurements. Repeatability and reproducibility of the measurement process are further assured through well documented test procedures and traceability of all measurements to national references.

Most ballistic test facilities use commercial-off-the-shelf (COTS) measurement systems to report projectile velocities. Unfortunately, few measurement systems provide uncertainty bounds on the performance of their devices, and even fewer present their uncertainty in a manner consistent with the *International vocabulary of metrology* (VIM) [2] and the *Guide to the expression of uncertainty in measurement* (GUM) [3] promulgated by the International Organization for Standardization (ISO). Therefore, the interpretation of the listed uncertainty may be inconsistent and could correspond to different assumptions on data distribution including normal distributions (i.e., standard deviations), triangular distributions (i.e., strict limits), or rectangular distribution (i.e., equal/uniform probability). This is a challenging obstacle for test facilities who are looking to understand and properly characterize the uncertainty related to their ballistic velocity measurement system.

In most metrological applications, the certainty of a quantity can be increased by repeating the measurement multiple times and assessing the mean response. In practice, however, the destructive nature of ballistics testing prevents this method from being applied. Further, the velocity of each shot is independent, therefore the certainty of a measured velocity cannot be increased by measuring the velocity of additional shots. To address the inherent variability in shot velocity seen in ballistics testing, common performance standards stipulate velocity certainty bounds for establishing fair tests. For example, a common standard for assessing the ballistic resistance of body armour (NIJ 0101.06) [4] requires a combined uncertainty of ±1.0 m/s (3.3 ft/s) on velocity measurement instrumentation. To limit the effects of uncertainty in experiments, redundant velocity measurements are also required for NIJ 0101.06 [4] and ASTM E3062 [5], with both measurements to be within 3.0 m/s (10 ft/s) of one another. Therefore, to adhere to the requirements of these test standards, a ballistics testing facility must be able to demonstrate the combined compliance of the measurement device and measurement process.

Paulter [6] outlined several key factors that contribute to uncertainty in ballistic velocity measurement devices. Additionally, the work by Riley [7] looked at the uncertainty applied to light-screen based devices which are the type most used in ballistics testing. Generally, two pairs of light screens are used to redundantly measure the average projectile velocity along two overlapping trajectories. In principle, these systems measure the time of flight between light screen pairs having a known separation distance, thereby permitting the velocity to be computed. These uncertainty analyses were expanded and demonstrated with Biokinetics' SpeedTube™, a high-definition light-screen based chronograph system, Figure 1, using a process that complies with ISO 17025 [1]. The SpeedTube™ consists of two sets of light screens that are rigidly mounted to one another using precisely machined components. Two categories of velocity uncertainty contributors were taken into consideration: 1) the uncertainty in time, and; 2) the uncertainty in distance. An analysis and quantification of potential contributors for the distance between light screens and time measurement was undertaken for the SpeedTube™ as depicted in TABLE I.

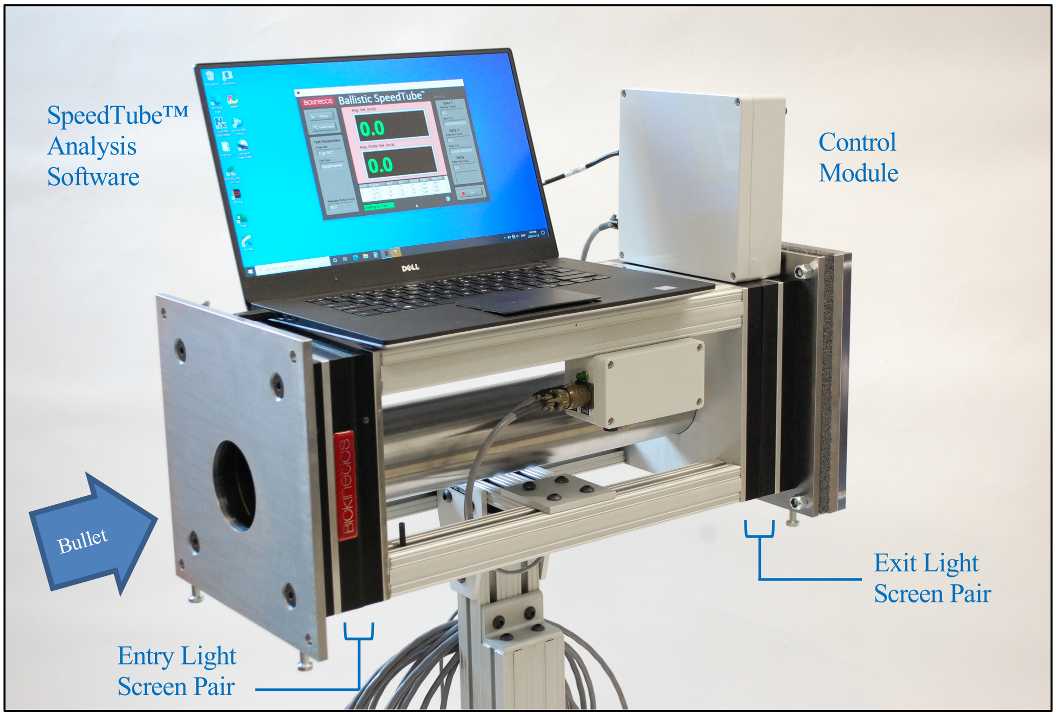

Figure 1: The SpeedTube™ ballistic chronograph

LIGHT SCREEN CHRONOGRAPH UNCERTAINTY

In practice, the most significant source of measurement uncertainty is that of the sampling process. This is generally quantified by performing a series of repeated measurements on the same measurand. For example, the width of a block could be measured three times with a calibrated measurement tool such as a caliper. There will be inherent variability between the different measurements – this is the sampling process uncertainty (or Type A uncertainty). The full uncertainty budget for a measurement device must also include Type B uncertainties which are derived using non-statistical means, such as external calibration sheets or assumed distributions.

TABLE I: UNCERTAINTY CONTRIBUTORS

Uncertainty in Distance	Uncertainty in Time
Caliper calibration	Counter calibration
Calibration resolution	Counter resolution
LVDT calibration*	Electrical conductivity
LVDT resolution*	
DAQ calibration*	
Thermal expansion	
Projectile yaw distance	
System alignment	
Light screen alignment	
Beam width*	
Light screen positioning *	
Extension rod length	
** Applies twice*	

The destructive nature of ballistics testing makes it impossible to perform repeated measurements under identical conditions of the projectile velocity. Therefore, the sampling (or process) uncertainty cannot be determined with a single measurement, i.e. n=1. As a result, the required uncertainty budget must include all Type B contributors and must propose methods of approximating Type A contributors.

As mentioned previously, there is no indication in ballistic standards as to what factors must be included when ensuring compliance with their stated velocity measurement requirements (such as ±3 m/s in NIJ 0101.06 [4]). There are many possible interpretations of this requirement. For example, perhaps the intent is that no measurements should ever show more than ±3 m/s from the actual velocity which can only be achieved with a given level of confidence and not 100% certainty when a statistical method is used. Or, should the stated uncertainty only include Type A uncertainties which must be inferred in ballistics testing. A third possible interpretation is that all possible uncertainty contributors in a well-constructed and justified uncertainty budget including a reasonable coverage factor should be within the stated velocity limits. A fourth option would be that if both redundant sets are within 3 m/s, then the velocity is "good" although there is no reference to the actual velocity. Although this interpretation ensures a high-precision device, it does not guarantee an accurate system (e.g. both measurements could be within 3 m/s of each other but offset from the true velocity).

The purpose of this paper is threefold: 1) to educate industry, researchers, and standard-makers as to the need for clear language consistent with the VIM: *International vocabulary of metrology* [2] guidelines, and the GUM: *Guide to the expression of uncertainty in measurement* [3]; 2) to demonstrate the steps required for the development of an uncertainty budget on a light screen-based ballistic velocity measurement device with the same characteristics as the SpeedTube™, and; 3) to apply this procedure to a COTS ballistic chronograph, the SpeedTube™ and arrive at an uncertainty budget. Note that other contributing factors to uncertainty not addressed in this analysis may need to be considered for different light screen-based ballistic velocity measurement systems to determine a relevant uncertainty budget. A numbers-based justification should be used in all cases to include or exclude possible contributors when applying these principles to a different chronograph.

The following equations and derivations apply to velocity measurement systems comprised of two parallel light screens where the bullet trajectory is expected to be orthogonal to the screens. For simplicity, the light screens will initially be referred to as planes (with zero thickness). A system includes a pair of light screens connected to a timing device that produces a single velocity measurement. A redundant system comprises two pairs of light screens and performs two velocity measurements. A ballistic chronograph computes the velocity by dividing the distance between the two light screens by the time period between light screens. The timing is typically determined by the output of a trigger signal which is based on a threshold being exceeded such as from the leading edge of the voltage signal as the projectile obscures each light screen. The governing equation is therefore:

$$V = \frac{d}{t} \tag{1}$$

Because the uncertainties of the distance and time terms are independent, they are individually quantified before being propagated to determine the uncertainty in velocity.

Quantification of Uncertainty in Distance

The distance between the light screens, d, is a parameter that would be manually entered into software and divided by the time required for the projectile to travel the distance between the light screens. It is important, however, to acknowledge that there is uncertainty in this measurement (i.e. the theoretical trajectory may deviate from the actual trajectory) based on the following factors:

1. Measurement of distance between the two light screes (Type A and Type B).
2. Light screen thickness - position of bullet tip relative to the light screen when triggered (Type A and Type B).
3. System alignment with respect to projectile trajectory (Type B).
4. Light screen alignment relative to the other screen in the system (Type B).
5. Thermal expansion of materials between the light screens (Type B).
6. Projectile yaw (Type B).

MEASUREMENT BETWEEN LIGHT SCREENS

The distance between the light screens must be accurately measured using a method suitably representative of the physical process. Because light screens are typically positioned far apart to increase the desired operational velocities and resolution (from counter limitations), it can be challenging to measure the light screen separation with accuracy. To increase the reliability of distance measurements, the approach taken herein directly measures the distance between the screens rather than between housings. Therefore, for the SpeedTube™, the distance was repeatedly measured with a high-precision sensor (e.g. linear variable displacement transducer / LVDT) according to the following process:

1. Rigidly mount a LVDT with a 12.7 mm diameter rod to the front of the SpeedTube™.
2. Connect the LVDT to the SpeedTube™ DAQ (data acquisition) card.
3. Rapidly propel the LVDT and attached rod into the SpeedTube™ (while recording the output).
4. Repeat Step 3. (5 times) – each test will provide a LVDT distance at which the first light screen triggered.
5. Measure the length of an extension rod (5 times). The extension rod, which also has a diameter of 12.7 mm, is then installed on the end of the LVDT rod to offset the measurement point to reach the last light screen (this is required because the LVDT does not have adequate range to cover the full distance between light screens).
6. Repeat Steps 3-4 with the extension rod installed to get the position of the second light screen.

The diameter of the rods was intentionally selected to be large such that the rising pulse duration be as short as possible, thus increasing the confidence in the baseline measured light-screen positions. Uncertainty in the position at which a light screen is triggered by a projectile is further discussed in the light screen thickness section. The distance between the two light screens is calculated using the measured values:

$$d = \left(\overline{d_2} + \overline{l_{ext}}\right) - \overline{d_1} \tag{2}$$

Where the distance between light screens, d, is a function of the measured positions of light screens one and two, $\overline{d_1}$, and $\overline{d_2}$, respectively, and the length of the extension rod, $\overline{l_{ext}}$. Each term has associated uncertainties which include sampling uncertainty (standard deviations), measurement device calibration certificates, and measurement resolutions. The following equations were found to represent the uncertainties of each term:

$$u_{\overline{d_1}} = \sqrt{u^2_{LVDT_{resolution}} + u^2_{LVDT_{calibration}} + u^2_{DAQ_{cal}} + s^2_{\overline{d_1}}} \tag{3}$$

$$u_{\overline{d_2}} = \sqrt{u^2_{LVDT_{resolution}} + u^2_{LVDT_{calibration}} + u^2_{DAQ_{cal}} + s^2_{\overline{d_2}}} \tag{4}$$

$$u_{\overline{l_{ext}}} = \sqrt{u^2_{caliper_{resolution}} + u^2_{caliper_{calibration}} + s^2_{\overline{l_{ext}}}} \tag{5}$$

In these equations, and throughout this paper, uncertainties are denoted by $u_{<description>}$, where the description indicates the source of the uncertainty. Experimentally determined uncertainties, quantified as the standard deviation of a series of measurements, are denoted as $s_{<measurand>}$, where the measurand is the property measured. The secondary measurement devices required to make these measurements (e.g. DAQ, LVDT, and caliper) were all calibrated by accredited ISO 17025 [1] facilities to ensure full traceability in the uncertainty process. The sample standard deviations are Type A errors while the calibration and resolution terms are Type B contributors. The magnitudes of the uncertainty terms listed above were found to be the following for the SpeedTube™:

$$u_{\overline{d_1}}, u_{\overline{d_2}} \approx 100\text{x}10^{-3}\ \text{mm} \tag{6}$$

$$u_{\overline{l_{ext}}} \approx 50\text{x}10^{-3}\ \text{mm} \tag{7}$$

LIGHT SCREEN THICKNESS

For most of the analysis process, the light screens are referred to as planes with inherent zero thickness which is a fair assumption in the derivation of other uncertainty sources. In practice, however, the light screens have finite thickness. At some point through the thickness of the light screen, the projectile tip will trigger a signal indicating that the light screen has been reached. There is, therefore, uncertainty associated with the beam width of the light screen. This is of particular relevance because it is the only term that applies twice; when the projectile reaches the first screen, and again when it reaches the second screen. As a first approximation with a beam width of 1 mm, a uniform probability is assumed indicating that no prior knowledge exists on the exact position causing the sensor to trigger. As per the *Guide to the expression of uncertainty in measurement* (GUM) Section (4.4.4) [3], a uniform probability distribution on the range of ±a has an associated uncertainty of $a/\sqrt{3}$. For uniform probability distributions, therefore, the beam width uncertainty is:

$$u_{beam} = \frac{0.5 \text{ mm}}{\sqrt{3}} \quad (8)$$

$$u_{beam} \approx 300\text{x}10^{-3} \text{ mm} \quad (9)$$

Given that the magnitude of this uncertainty term was deemed to be unacceptably high, efforts were made to refine the estimate of uncertainty for this term. An experiment, where a yawed 0.357 Magnum bullet was obliquely mounted to the end of the LVDT rod used in the previous section, was conducted. The bullet and LVDT rod were repeatedly pushed through the light screen. The rod was also rotated through several positions to account for possible blind spots in the light screen. In the end, the uncertainty was determined to be:

$$u_{beam} \approx 100\text{x}10^{-3} \text{ mm} \quad (10)$$

Where the beam width uncertainty relied on a similar process to the evaluation of the initial light screen positions, therefore, the uncertainty equation is equivalent.

$$u_{beam} = \sqrt{u^2_{LVDT_{resolution}} + u^2_{LVDT_{calibration}} + u^2_{DAQ_{cal}} + s^2_{\overline{beam}}} \quad (11)$$

Where the beam width uncertainty, u_{beam}, is a function of the uncertainties in the LVDT calibration and resolution, $u_{LVDT_{resolution}}$ and $u_{LVDT_{calibration}}$, respectively, the data acquisition calibration, $u_{DAQ_{cal}}$, and manual measurements of the beam positioning, $s_{\overline{beam}}$.

SYSTEM ALIGNMENT

Proper alignment of the system is required to accurately compute the velocity in a light screen chronograph. The two light screen planes are assumed to be parallel to one another, and under ideal conditions, the bullet trajectory axis would be orthogonal to both planes. There are practical limitations to consider when quantifying the uncertainty of system misalignment. For example, the SpeedTube™ implements front and rear endcaps that mount parallel to each light screen at the entrance and exit of the device. These opaque endcaps have pinholes that only allow a bore mounted laser beam to pass all the way through to the target if the system is adequately aligned. The exact misalignment angle cannot be known with this method, however, it is reasonable to assume a uniform probability distribution within the range of angles that result in the laser beam shining through the pinholes. The expected system misalignment, $\theta_{alignment}$, becomes:

$$\theta_{alignment} = \frac{\text{atan}\left(\frac{\emptyset_{pinhole}}{d_{endcaps}}\right)}{\sqrt{3}} \quad (12)$$

Which is a function of the pinhole diameter, $\emptyset_{pinhole}$, and the endcap separation distance, $d_{endcaps}$. For the SpeedTube™, this angle evaluates to approximately 0.1°. This misalignment also increases the distance that the projectile must travel between the two light screen planes. The actual distance travelled is:

$$d_{alignment} = \frac{d}{\cos(\theta_{alignment})} \tag{13}$$

The corresponding uncertainty is:

$$u_{alignment} = d(\sec(\theta_{alignment}) - 1) \tag{14}$$

The SpeedTube™ contains two independent pairs of light screens nested within each other. Therefore, the distance between the two outer screens is taken as a worst-case approximation to quantify this uncertainty contributor. The uncertainty from misalignment of the system is found to be in the order of:

$$u_{alignment} \approx 800\text{x}10^{-6}\ \text{mm} \tag{15}$$

LIGHT SCREEN PAIR DEVIATION FROM PARALLEL

After examining the overall system alignment, the assumption of parallel light screen planes should be considered. For example, if the first light screen is orthogonal to the projectile trajectory but the second is not (and is deviated by angle θ_{screen}), the distance travelled by the projectile may be increased or decreased depending on the dispersion of shots around the intended shot placement indicated by the bore laser. The angle θ_{screen} is a function of manufacturing tolerances and assembly procedure. Although this value is expected to be quite small, it must be considered for a complete uncertainty budget. To quantify this uncertainty source, it is assumed that a projectile passes through an axis offset from the central axis by distance $d_{dispersion}$, that the second light screen is at angle θ_{screen} with respect to the first light screen, and that the distance between the two light screens along the central axis is *d*. The projectile must travel a distance of:

$$d_{parallel} = d + d_{dispersion} \tan(\theta_{screen}) \tag{16}$$

The uncertainty term was simplified to the following:

$$u_{parallel} = d_{dispersion} \tan(\theta_{screen}) \tag{17}$$

To quantify this error, a worst-case deviation of 1° with a uniform distribution was assumed. The expected dispersion was determined from a set of 10 shots with a 2-grain RCC projectile where the typical deviation from the central axis was 12.3 mm. The 2-grain RCC projectile was selected because it is typically associated with very high dispersion compared to other more typical projectiles (e.g. 9 mm, 7.62 mm, etc.), thus providing a conservative estimate of uncertainty. Under these conditions, the uncertainty in distance caused by non-parallel light screens is in the order of:

$$u_{parallel} \approx 120\text{x}10^{-3}\ \text{mm} \tag{18}$$

THERMAL EXPANSION/CONTRACTION

A typical ballistic standard may require ambient temperature conditions to be within a range of 15°C-25°C. Over that range of temperatures, any materials used to mount the screens to one another may expand or contract, increasing or decreasing the distance the projectile must travel between the two light screens. A conservative approach for quantifying this error is assuming a uniform distribution within possible operating range as environmental conditioning systems would likely maintain a mid-range temperature of 20°C. From the assumption of uniform distribution, the expected temperature variability is then:

$$\Delta T = \pm \frac{5°\text{C}}{\sqrt{3}} = \pm 2.89°\text{C} \tag{19}$$

The uncertainty in distance between the screens over the operating range is then a function of the lengths, l_i, and thermal expansion coefficients, α_i, of the i components separating the two light screens.:

$$u_{thermal} \approx \sum_{i=1}^{n} \alpha_i l_i \Delta T \tag{20}$$

This uncertainty is affected by the distance between light screens and coefficient of thermal expansion (both have positive correlations). Therefore, the materials selected during the design phase should have low thermal expansion coefficients to minimize the associated uncertainty. The length between light screens is also an important factor; however, the relationship is less clear due to other uncertainties (e.g., system alignment) being inversely correlated with the distance. For the SpeedTube™, this error is approximately:

$$u_{thermal} = 70 \text{ x}10^{-3}\text{mm} \tag{21}$$

YAW CYCLE

The ballistic velocity is best represented by the time the projectile centre-of-gravity (CG) passes the plane of the first light screen to the time it passes through the second light screen. In practice however, the tip of the bullet is the first point to pass through the light screen, triggering the signal. Therefore, the position of the tip is what is used to determine the target velocity with the tip being sensitive to positional variance from the yaw cycle. For example, if the projectile has 0° of yaw as it passes through the first light screen, the distance between the CG and the light screen plane is l_{CG-TIP}. Now, if the projectile has a yaw of θ_{yaw} as it passes through the second light screen, the distance between the CG and light screen plane is $l_{CG-TIP}\cos(\theta_{yaw})$. Therefore, the total distance travelled by the CG of the projectile between the times the two light screens are triggered is:

$$d_{actual} = d + \left(l_{CG-TIP} - l_{CG-TIP}\cos(\theta_{yaw})\right) \tag{22}$$

The contributions from projectile yaw towards the uncertainty in distance between light screens is therefore:

$$u_{yaw} = \left(1 - \cos\left(\theta_{yaw}\right)\right) * l_{CG-TIP} \tag{23}$$

A conservative quantification of this uncertainty would be to use a 7.62 mm projectile (l_{CG-TIP}=17 mm) and assuming the yaw changes by θ_{yaw} =5° between the two light screens. Application specific values could be determined based on the projectile-specific yaw cycles; however, conservative estimates must be used where insufficient practical information exists to quantify an uncertainty source. For the 7.62 mm specifications listed above, the uncertainty is in the range of:

$$u_{\text{yaw}} = 60\text{x}10^{-3}\ \text{mm} \tag{24}$$

Quantification of Uncertainty in Time

The destructive nature of ballistics testing is such that there is only one opportunity to measure the time the projectile travels between light screens. The primary factors used to evaluate the uncertainty in time were instrument calibration (the counter was calibrated by an ISO 17025 accredited facility to ensure traceability), counter resolution and electrical propagation delays. The time is internally computed by the counter according to the following equation:

$$\text{t} = \frac{n}{f} \tag{25}$$

Here, n is the number of ticks recorded by the counter, and f is the frequency of the counter. According to the *Guide to the expression of uncertainty in measurement* (GUM) Section 5.1 [3], for uncorrelated uncertainties, the combined uncertainty is a function of partial derivatives with respect to each contributor and the associated uncertainty:

$$u_c^2(y) = \sum_{i=1}^{N} \left(\frac{\partial f}{\partial x_i}\right)^2 u^2(x_i) \tag{26}$$

Therefore, the uncertainty equation for the time measurement is:

$$u_t = \sqrt{\left(\frac{1}{f}u_n\right)^2 + \left(-\frac{n}{f^2}u_f\right)^2} \tag{27}$$

The uncertainty related to electrical propagation delays was determined to be orders of magnitude smaller than the counter resolution and was excluded from the analysis. The justification for exclusion relied partially on the following: the SpeedTube™ is a differential system and, therefore, because any delays within the circuitry are expected to be very similar between light screens 1 and 2, the simple subtraction of the two counter times to determine the total time, t, cancels out much of this uncertainty. As further justification, assuming one circuit had the equivalent of 10 cm more wiring, the propagation uncertainty is still at least two orders of magnitude smaller than the uncertainty from the resolution of the counter. If the system used longer wires for one light screen than the other, this difference may have to be considered. The equation for uncertainty in time contains a non-constant value: n. Two approaches could be used to quantify this value: 1) a velocity-dependent table could be generated, or; 2) a reasonable estimate for the worst-case value could be used. Here, the latter approach is taken, where the number of ticks n is related to the velocity V, light screen separation d and counter frequency f:

$$n = \frac{\mathrm{d}}{V} f \tag{28}$$

Substitution of this expression for n into the uncertainty equation for time yields:

$$u_t = \sqrt{\left(\frac{1}{f} u_n\right)^2 + \left(-\frac{d}{Vf} u_f\right)^2} \tag{29}$$

After inserting parameters specific to the SpeedTube™, and assuming a velocity that maximizes the uncertainty (e.g. 300 m/s), the uncertainty in time is approximately:

$$u_t \approx 4\text{x}10^{-9}\ \text{s} \tag{30}$$

Propagation of Uncertainty for Velocity

As previously stated, the fundamental equation for the velocity is:

$$V = \frac{d}{t} \tag{1}$$

As per GUM Section 5.1 [3], the combining of uncorrelated uncertainties leads to the following combined uncertainty equation (after substitution of the partial derivatives of equation 1).:

$$u_V = \sqrt{\left(\frac{u_d}{t}\right)^2 + \left(-\frac{d}{t^2} u_t\right)^2} \tag{31.1}$$

Which, after substitution to replace the time variable, becomes:

$$u_V = \sqrt{\left(\frac{V}{d} u_d\right)^2 + \left(\frac{V^2}{d} u_t\right)^2} \tag{31.2}$$

The relationship is a function of the distance between light screens, d, projectile velocity, V, and the overall uncertainties in distance and time, u_d and u_t, respectively. Before computing this value, it is necessary to combine all distance-related uncertainty terms into one value: u_d. The equation for distance travelled between light screens 1 and 2 is:

$$d = \left(\overline{d_2} + \overline{l_{ext}}\right) - \overline{d_1} + \delta \tag{32}$$

The added term, δ, represents all the other possible contributors causing an increase of decrease in distance between light screens as described in the previous section. All uncertainty contributors for the distance can be summed (as variances) to determine the combined uncertainty.

$$u_\delta = \sqrt{2u_{beam}^2 + u_{alignment}^2 + u_{parallel}^2 + u_{thermal}^2 + u_{yaw}^2} \qquad (33)$$

$$u_\delta \approx 210\text{x}10^{-3} \text{ mm} \qquad (34)$$

Therefore,

$$u_d = \sqrt{u_{\overline{d_1}}^2 + u_{\overline{d_2}}^2 + u_{\overline{l_{ext}}}^2 + u_\delta^2} \qquad (35)$$

$$u_d \approx 260\text{x}10^{-3} \text{ mm} \qquad (36)$$

For the range of velocities expected in typical ballistics testing, the uncertainty evaluates to the values shown in TABLE II. The uncertainty values shown in the table are standard uncertainties (i.e., ±σ).

As discussed previously, this is one possible interpretation of the uncertainty given in ballistic standards. However, these uncertainty values are inconsistent with the requirements of GUM [3]. Rather, an expanded uncertainty accounting for a specified level of confidence is required for standard uncertainty reporting.

Expanded Uncertainty for Velocity

The expanded uncertainty is defined as the product of the standard uncertainty by a coverage factor, k, such that the listed uncertainty is:

$$U_V = ku_V \qquad (37)$$

TABLE II: PROPAGATED VELOCITY UNCERTAINTY

V (m/s)	u_V (m/s)
300	0.18
400	0.24
500	0.30
600	0.36
700	0.42
800	0.48
900	0.54
1000	0.60
1100	0.65
1200	0.71
1300	0.77
1400	0.83
1500	0.89
1600	0.95
1700	1.01
1800	1.07

In general, a value of k≈2 corresponding to a level of confidence of approximately 95% is recommended. The coverage factor is based on a t-distribution to achieve a sufficient level of confidence for the application, therefore, it is necessary to determine the effective number of degrees of freedom before determining the correct coverage factor. The Welch-Satterthwaite equation must be used to determine the effective number of degrees of freedom.

$$v_{eff} = \frac{u^4}{\sum_{i=1}^{N} \frac{u_i}{v_i}} \tag{38}$$

For this equation, all i sources of uncertainty, u_i, and their relevant degrees of freedom, v_i, must be included in order to compute the overall effective number of degrees of freedom from the overall uncertainty, u. According to GUM (G.4.3) [3], it is not unreasonable to select $v_i \to \infty$ for Type B errors as the standard methods involve providing conservative estimates that are unlikely to be exceeded, therefore, only Type A errors must be considered.

$$v_{eff} = \frac{u_v^4}{\frac{s_{\overline{d_1}}^4}{v_{s_{\overline{d_1}}}} + \frac{u_{s_{\overline{d_2}}}^4}{v_{s_{\overline{d_2}}}} + \frac{u_{s_{\overline{l_{ext}}}}^4}{v_{s_{\overline{l_{ext}}}}} + 2\frac{u_{beam}^4}{v_{u_{beam}}} + \frac{u_{parallel}^4}{v_{u_{parallel}}}} \tag{39}$$

In this example, the effective degrees of freedom become:

$$v_{eff} \approx 160 \tag{40}$$

The effective number of degrees of freedom is used with the intended coverage level using the *t*-statistic to determine coverage factor. Therefore, the coverage factor for a confidence level of approximately 95% with 160 degrees of freedom is k=2.02. The expanded uncertainty for the SpeedTube™ is shown in tabular and graphical forms in TABLE III and Figure 2.

TABLE III: EXPANDED VELOCITY UNCERTAINTY

***V* (m/s)**	***U* (m/s)**
300	0.36
400	0.48
500	0.60
600	0.71
700	0.83
800	0.95
900	1.07
1000	1.19
1100	1.31
1200	1.43
1300	1.55
1400	1.67
1500	1.79
1600	1.91
1700	2.02
1800	2.14

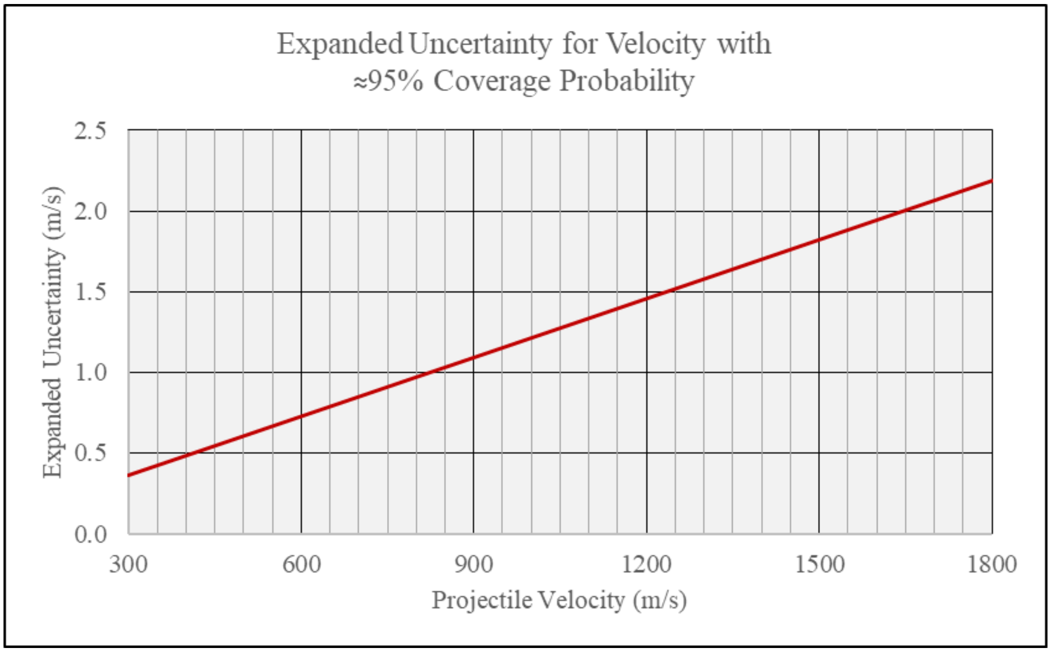

Figure 2: Expanded uncertainty for the SpeedTube™ with approximately 95% coverage probability corresponding to a coverage factor of k≈2.

As can be noted, the expanded uncertainty changes with projectile speed but is typically ±0.12% of the test velocity for the SpeedTube™.

DISCUSSION

The various sources of uncertainty identified and quantified in this analysis were combined using methods described in GUM [3]. A coverage factor of k≈2 was then applied to report the uncertainty in velocity measurement in a manner consistent with common practices. This factor corresponds approximately to a t-value confidence interval of 95%. The uncertainty of the SpeedTube™ was found to be normally distributed with an uncertainty of approximately 0.1% of shot velocity with a coverage factor of k≈2 over the range of 300 m/s to 1800 m/s. If the desired coverage factor is k≈1 (i.e. corresponding to a 68% confidence interval), the uncertainty becomes 0.06% for the SpeedTube™.

For reference, the maximum velocity specified in NIJ 0101.06 [4] is approximately 900 m/s (although individual shot velocities could be higher for V_{50} testing). With the previously mentioned requirement of ±1.0 m/s combined uncertainty on velocity measurements, this would correspond to just over 0.11% at 900 m/s. However, insufficient information is provided to indicate what factors should be considered in determining the relevant "combined uncertainty", and there is no mention of the required coverage factor. It is critical that the basis for assessing uncertainty is properly known by researchers, manufacturers, and service providers to obtain meaningful data on the ballistic performance of armour systems.

CONCLUSION

It was shown that for the SpeedTube™, a ballistic light-screen based chronograph, the overall uncertainty of the system must incorporate the individual uncertainties associated with the physical attributes of the light screens as well as the electronics used for monitoring and evaluating the occlusion from the projectile and determination of the projectile velocity. It was further shown that each attribute such as light screen separation needs to account for uncertainties in the methods used for measuring distances, the identification of the true physical location of occlusion, the relative light screen alignment, environmental temperature effects, and the typical state of projectile (e.g. yaw and dispersion). Similarly, the parameters associated with timing and velocity determination included the light screen triggering mode, the signal sampling speed and variability of the sampling frequency. The relative contributions to uncertainty that were considered for the SpeedTube™ are presented in Figure 3. This can help identify key sources of uncertainty and potential for future system improvements including functional, operational, and procedural in nature.

The process used to develop an uncertainty budget relies on experience-based judgement and numbers-driven decision-making to justify which factors should be included and how these contributors be quantified. Where it is either impossible or impractical to quantify exact values, sensible assumptions are used to simplify the problem such that a reasonable representation of the uncertainty be determined. In these cases, conservative approaches are required as to over-estimate the uncertainty rather than erroneously under-report the contributor.

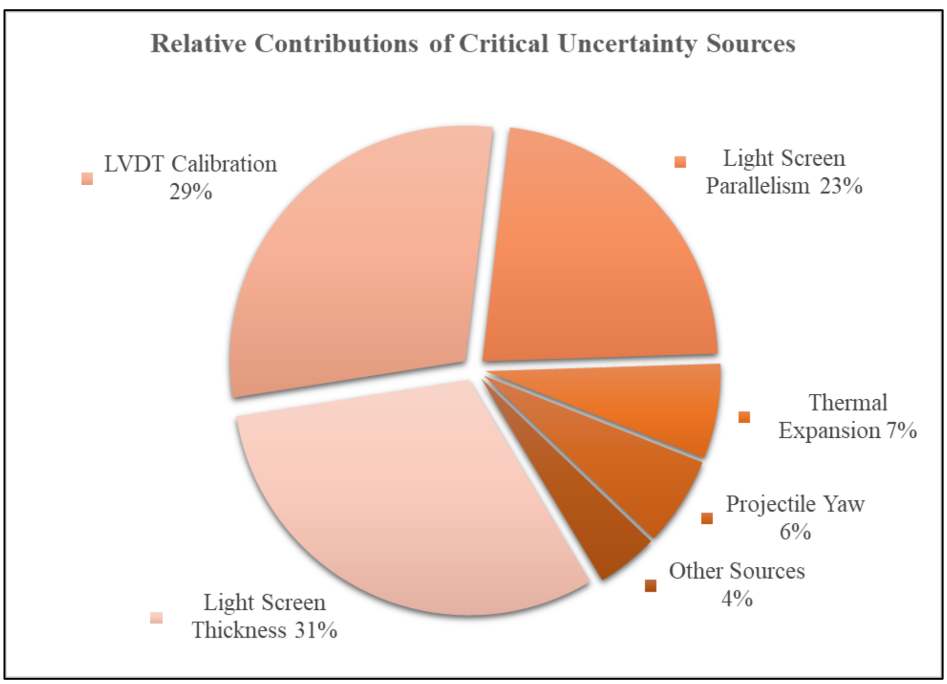

Figure 3: Relative contributions to combined uncertainty in velocity measurements for the SpeedTube™ including all factors identified and quantified in this analysis.

The limitations of this analysis include the following factors:

1. The uncertainty budget developed herein for the SpeedTube™ relied on many assumptions and simplifications that could affect the accuracy of reported values. Where possible, to draw results from a limited number of projectile types and operating conditions, analysis parameters were selected to ensure conservative estimates for each contributor. For example, the worst-case change in yaw between the light screen pair was assumed to be 5° based on standards, such as NIJ 0101.06 [4], which state that the projectile yaw must be within this value of the intended angle upon impact. Although this does not mean that the projectile yaw cannot exceed 5° anywhere along the trajectory, physically, the projectile has a predictable yaw cycle between the muzzle and target, at least for non-tumbling stabilized projectiles. These yaw cycles make it extremely unlikely that a 5° change in resultant yaw between the two light screens would occur. Other systems that have larger gate separation may have a larger associated uncertainty.
2. The alignment uncertainty assumes that the projectile follows the ideal trajectory. In practice, dispersion of the projectiles about this trajectory could increase the uncertainty. However, the dispersion of the projectiles at impact is likely much greater than the dispersion at the chronograph due to its proximity to the muzzle.
3. Different projectiles were used at different stages of the analysis to always report a worst-case scenario for each uncertainty contributor. For example, when considering yaw, a relatively long 7.62 mm projectile was used, but when considering light screen parallelism, a 2-grain RCC was used because it typically has higher dispersion. This approach was used to increase the generalization of the uncertainty analysis to reasonably represent a variety of projectiles in different operating conditions. Further, the study of the light-screen thickness was only performed for a bullet; an RCC or and FSP would likely have a different response. Specifically, changes in the roll of an FSP with a leading chamfer edge could result in a different part of the fragment triggering the light screen. This effect would affect the uncertainty due to yaw in all light screen-based chronographs.
4. This analysis was performed using the specific parameters and equipment of the SpeedTube™. The results are therefore specific to this device and cannot be used as a representation of general light-screen based chronographs. The approach, however, can be used as a basis to determine the uncertainty budget of similar devices having similar characteristics.
5. The SpeedTube™ is an integrated redundant chronograph, therefore it has two pairs of light screens with different spacing that independently measure the velocity along the flight trajectory. Rather than reporting two separate uncertainty values for the systems two pairs of light screens, the worst-case approach was followed. If an uncertainty contribution was proportional to the light screen separation, the larger spacing was used to provide the most conservative estimate. Similarly, if the uncertainty contribution was

inversely proportional to the light screen spacing, the smaller spacing was used. The sampling system for each light screen pair of the SpeedTube™ is also independent with asynchronous timing and counting circuits used.

6. All contributors analyzed were assumed to occur independently of one another. In practice, it is possible that multiple worst-case conditions would not occur at the same time or would not occur alone. For example, the system alignment uncertainty (Equation 14) would never increase the distance between light screens and is therefore more accurately represented by a +0/-$u_{alignment}$ rather than $\pm u_{alignment}$.

Despite the previously listed limitations to the present study, the uncertainty budget detailed herein is a reasonable, conservative, approximation of the expected measurement uncertainty for the SpeedTube™. It was demonstrated using a process outlined by the *Guide to the expression of uncertainty in measurement* (GUM) [3] and using a vocabulary consistent with *International vocabulary of metrology* (VIM) [2], that including possible contributors related to the distance between light screens, operating conditions and test parameters, and measured time lead to a system that largely meets even the most stringent interpretation of the NIJ 0101.06 [4] ballistic testing standard.

The ambiguity of the uncertainty requirements in common ballistic standards with respect to accuracy limits (i.e. distribution), factors to consider, and coverage factors makes it difficult for researchers, engineers, and manufacturers to interpret results and design equipment. For this reason, the authors strongly suggest that the *International vocabulary of metrology* (VIM) [2] be consulted by all persons developing testing standards with uncertainty constraints on the measurements. Even with the extremely conservative approach followed in this analysis, including specifying a coverage probability of approximately 95% and including all possible factors, the uncertainty was found to be just over 0.1% of test velocity. Therefore, the SpeedTube™, a COTS ballistic chronograph was demonstrated to meet the requirements ±1 m/s stipulated in NIJ 0101.06 [4] up to velocities over 800 m/s while simultaneously satisfying the requirement for a redundant velocity measurement system.

To this end, the identification and quantification of factors contributing to ballistic velocity measurement uncertainty has been carried out and applied to a commercial light-screen system demonstrating the uncertainty assessment process in practice with widespread application to similar devices used in most ballistic test facilities but it must be recognized that additional or different factors may need to be addressed for each measurement system.

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